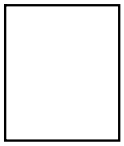
Warmup!
Solve by completing the square.
1. $2x^2 + 8x - 3 = 0$
2. $x^2 + x - 5 = 0$



6-5 The Quadratic Formula and the Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Complete the square. $\left(\frac{b}{2a}\right)^2$

$ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$

$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$

$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

ex 1
$3x^2 + x - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2-4(3)(-1)}}{2(3)}$

$\left\{\frac{-1 \pm \sqrt{13}}{6}\right\}$

ex 2
$5x^2 + 8 = -12x$

$5x^2 + 12x + 8 = 0$

$x = \frac{-12 \pm \sqrt{144 - 4(5)(8)}}{2(5)}$

$\frac{-12 \pm \sqrt{-16}}{10}$

$x = \frac{-12 \pm 4i}{10}$

$\left\{\frac{-6 \pm 2i}{5}\right\}$

The Discriminant

$D = b^2 - 4ac$

$x = \frac{-b \pm \sqrt{D}}{2a}$

Determines the nature of the roots.

Three Cases

I. D > 0 (positive) 2 ℝ roots

II. D = 0 ℝ double root

III. D < 0 (negative) 2 imaginary roots

Ex
$x^2 - 8x + 5 = 0$

$D = b^2 - 4ac$
$64 - 4(1)(5)$
44
2 ℝ roots
irrational

Also able to determine if the roots are rational or irrational.

Rational
a, b, & c must be rational and D must be a perfect square (Real)

ex
$x^2 + 10x + 25 = 0$

$D = 100 - 4(1)(25)$
$D = 0$
Double ℝ root + rational (a)

ex
$x^2 - 4x + 13 = 0$

$D = 16 - 4(1)(13)$
$16 - 52$
2 imaginary roots

Determine as much as you can about the roots:

1. $y^2 - 3y - 1 = 0$ $9 - 4(1)(-1) = 13$ 2 ℝ irrat. root

~~2. $3a^2 - 10a = -11$~~

3. $5x^2 + 2\sqrt{10}x + 2 = 0$ $(2\sqrt{10})^2 - 4(5)(2)$ 0 Double ℝ irrational

~~4. $3b^2 = 14b + 24$~~

Find the value for k such that there are 2 imaginary roots

ex
$5x^2 - 2x + k = 0$

Set $D < 0$

$4 - 4(5)k < 0$
$4 - 20k < 0$
$4 < 20k$
$k > \frac{1}{5}$

Find the value for k such that there is a double root

ex
$3x^2 + 2x + k = 0$

$D = 0$

$4 - 4(3)k = 0$
$4 = 12k$
$\frac{1}{3} = k$

p318
15-25 odd and 45a
(not #21)